

Real Estate Return Distributions with New NCREIF Data Series

by

Michael S. Young

1111 Asher Street, Jasper, TN 37347
phone: 805-931-9737 / e-mail: mike@msyapps.com

Roger J. Brown, Ph.D.

Colvin Institute of Real Estate Development
School of Architecture, Planning and Preservation
University of Maryland, College Park, MD 20742
phone: 619-929-4387 / e-mail: rjbr21@gmail.com

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Abstract: The accuracy of real estate return distribution parameter estimation is affected by the tools used to do the work as well as the data sets employed. Consistent with previous studies, investment risk models with infinite variance describe distributions of individual property returns in the new NCREIF Indicators: Capital Performance and Property Operations individual property database over the period 1990 to 2021. Applying Maximum Likelihood Estimation (MLE) to historic data shows real estate investment risk to be heteroscedastic, but the Characteristic Exponent of the investment risk function varies more among property types than previously reported whether computed by MLE or other estimation techniques.

Key words: Asset-specific risk, return distributions, Maximum Likelihood Estimation, non-Normality, diversification, institutional investing, NCREIF

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Introduction

The technology employed to define the shape of financial asset return distributions has been a persistent, if sporadic, effort for over fifty years. With each new attempt has come some improvement in precision and greater appreciation for just how far removed financial asset return distributions are from Gaussian Normal distributions, the default presumption in countless academic studies, educational courses, and applications.

Shortly after cave painting came double-log graph paper upon which Benoit Mandelbrot (1963) plotted cotton prices over a sixty-year period using a #2 lead pencil. Regardless of the

time interval he chose, the patterns always looked the same, namely a cyclical, non-periodic pattern with respect to scaling, but an erratic pattern with respect to a Normal distribution of price levels. Had the return distributions been Gaussian Normal, the line connecting the points would have been a straight line, but that was not to be. With departures from the straight-line expectation more pronounced at the high positive and low negative ends of the plot, Mandelbrot concluded that the distributions were not likely to be identically, independently distributed (iid) Normal but some other Levy-stable distribution with infinite or undefined variance.¹

While the fractal view of the world has been shown repeatedly to be a superior representation of the real world, the predominant view espoused in academia and among practitioners is that simplified models rooted in easily-taught and easily-comprehended models are “good enough.”

Being just “good enough” may suffice for teaching general principles, but it can fall woefully short of providing tools for practical applications of those principles. One case in point is the degree to which adding uncorrelated risky assets to a portfolio might reduce the non-systematic risk of the portfolio. Non-systematic risk reduction at the rate of *one over the square root of the number of assets* is commonly taught in investment classes and espoused by investment managers. However, research by Young and Graff (1995) has shown that a more accurate picture of commercial property return distributions implies non-systematic risk reduction at the *rate of about one over the cube root of the number of assets*.

These differences are non-trivial (as shown graphically in Exhibit 11a and b). Where it would take 100 assets whose returns were Normally distributed (i.e., having a Levy-stable Characteristic Exponent of 2.0) to affect a ten-fold reduction in non-systematic risk in a portfolio, it would take 1,000 assets to produce the same result for assets whose returns approximate the empirical results of studies by Young and Graff (1995) and Young (2008) (i.e., having a Levy-stable Characteristic Exponent of 1.5).

Based on the estimates using the MVI+FCFY series as its source having a Characteristic Exponent of approximately 1.4, it would require more than 3,100 assets to attain a ten-fold reduction in non-systematic risk in a commercial real estate asset portfolio. To put this into some perspective, in 2021, the entire MVI+FCFY series contained 8,732 properties across four property types for an estimated total market value in excess of \$700,000,000,000.

As in the prior work on real estate return distributions in Young and Brown (2012), this article uses the latest Maximum Likelihood Estimation (MLE) techniques and software, with confidence interval estimation from Nolan, both departures from earlier work in the US, Australia, the UK, and Germany.

However, this is the first research use of NCREIF’s new data series formally called “NCREIF Indicators: Capital Performance and Property Operations” that includes two related sets: the Market Value Index (MVI), the Free Cash Flow Yield (FCFY). We sum the individual MVI and the FCFY quarterly return statistics for each property and then chain-link four calendar quarter

sums to create annual total returns for commercial property from 1990 to 2021 disaggregated by four property types: Office, Retail, Industrial, and Apartment. Importantly, these series differ from the NCREIF Property Index (NPI) that has been used in all previous studies on domestic commercial real estate return distributions.

The major difference between the NPI and the MVI+FCFY series is that the NPI includes properties that, from time to time, incur substantial capital expenditures for renovation, expansion, partial sales or repurposing. These expenditures have a significant impact on total returns reported in the NPI both positive and negative.² More importantly, the properties incurring these capital expenditures change their physical, functional, or financial economic character as a result of the expenditure. In a real sense, these properties are not of the same character pre- and post- major capital expenditures or, in common real estate parlance, they are not “same store.”

The new NCREIF data series address this “same store” problem by applying filter rules to exclude properties and their computed results in quarters when major capital expenditures are incurred. Owing to accounting conventions, major capital expenditures can be both either positive or negative in amount and accordingly total returns in the NPI when these expenditures happen may increase or decrease substantially, often at rates that exceed 100% plus or minus in a single quarter. By removing quarters when this happens, the remaining quarterly total returns in the data set computed as the sum of market value change and distributable net cash flow (MVI+FCFY) seldom push the results beyond logical or realistic bounds and will be somewhat different than total returns in the classic NPI as will become clear later. It also helps confirm that heavy tails, when observed, are not the result of accounting anomalies.

Young, Fisher, and D’Alessandro (2017) provide a complete description of the NCREIF Indicators: Capital Performance and Property Operations data series, its development, and its uses. Also, in light of the unique filter rules employed and the fact that this article is the first published research utilizing the new data series, a section below will include an extensive descriptive quotation about the filter rule from this *Journal of Real Estate Literature* article.

Levy-stable Distributions in Real Estate

Starting in the early 1970s, the degree to which Modern Portfolio Theory (MPT) and Efficient Markets Hypothesis (EMH) came to dominate the investment world was as remarkable as it was contrary to traditional thinking about how theories may be derived from empirical evidence. After nearly two decades of influence in the stock market, MPT and EMH were introduced to real estate without empirical justification. In Young and Graff (1996) the authors assert: “MPT and EMH seem to have been introduced into real estate to justify the use of particular statistical techniques and portfolio strategies rather than as a consequence of empirical analysis of investment return and risk characteristics. In science, the situation is generally reversed: theories are developed to explain observations.”

In a departure from the facile presumption of Gaussian Normal distributions, McCulloch's quantile-based methodology (McCulloch (1986)) has proven robust for the analysis of real estate return distributions in the United States (Young and Graff (1995), Brown (2000), and Young (2008)), Australia (Graff, Harrington, and Young (1997)), the United Kingdom (Young, Lee, and Devaney (2006)), and, more recently, in Germany (Richter, Thomas, and Fuss (2011)).³

MLE has surpassed McCulloch's estimators in ease of use for both academics and practitioners alike because fast, inexpensive computing is now ubiquitous. MLE applications have been available in the Fortran programming language for two decades, ten years ago these routines were added to the kernel of Wolfram Research's symbolic computing software Mathematica.

The idea behind MLE of distribution parameters is to derive the various parameters which maximize the probability that the parameters best describe the sample distribution. While seemingly simple in concept, MLE is numerically intense, which makes the availability of powerful computers for data processing necessary.

For those inclined toward empirical analysis of sample data, the advent of powerful software coupled with the availability of fine-grained performance data offer an opportunity to (1) examine the real world characteristics of real estate as an investable asset class, (2) probe for similarities and differences among discrete physical and functional dimensions like property types or location, (3) develop strategies and tactics to take advantage of persistent similarities and differences, (4) test alternative measures of real estate risk that could be used to mitigate potential losses or enhance portfolio performance, and (5) finally abandon the mathematics of MPT altogether as a technical construct of finance that is unworkable in real estate.

For our purposes, perhaps the most useful byproduct of MLE's good statistical properties is its ability to provide confidence intervals to quantify uncertainty within the data. Compared to McCulloch's estimates of confidence, modern MLE measures are a substantial improvement in precision. In the results that follow, for example, the magnitude of the standard errors for the characteristic exponent from MLE are between one-quarter and one-third the size of those produced by McCulloch's quantile model.

By application of MLE as employed in Young and Brown (2012), this article tests whether more accurately stated property return distributions have finite variance and are Gaussian Normal. We also compare those results to the prior article for the overlapping period 1990 to 2010. The short answer is that they still are not Normal over the four major property types with over a decade of new data covering additional economic cycles. The longer answer confirms the same for the combined annual data series with equal precision and statistical confidence despite the differences in the number and individual property returns of properties within the new NCREIF Indicators: Capital Performance and Property Operations data compared to the classic NCREIF Property Index (NPI).

Levy-stable Distributions

Levy-stable distributions offer a number of useful properties. First, they are shape preserving under addition and linear transformation, comporting with a long held premise of the finance paradigm that prices are the result of an accumulation of randomly arriving information. This permits a host of useful results in mathematical statistical theory which relate to sums of random variables. Second, they appeal to the Generalized Central Limit Theorem which holds that if a distribution has a limiting distribution, it must be a member of the Levy-stable class. The Normal distribution is a special case of Levy-stable distributions as it has a finite variance.

The characteristic functions of Levy-stable distributions have the following form for cases where $\alpha > 1$:

$$\psi(t) = e^{i\delta t - |\gamma t|^\alpha [1 + i\beta(-1 + |\gamma t|^{1-\alpha}) \operatorname{sgn}(t) \tan(\pi\alpha/2)]} \quad (1)$$

The four parameters α , β , γ , and δ in Equation (1) completely characterize the distribution.

The Characteristic Exponent α lies in the half-open interval $(0, 2]$ and measures the rate at which the tails of the density function decline to zero. The larger the value of the Characteristic Exponent α , the faster the tails shrink toward zero. When $\alpha = 2.0$, the distribution is Normal.

While the means (first moments) of Levy-stable distributions with Characteristic Exponents $\alpha > 1.0$ do exist, variances (second moments) do not exist—i.e., are infinite—for those distributions with Characteristic Exponents $\alpha < 2.0$.

The Skewness Parameter β lies in the closed interval $[-1, 1]$, and is a measure of the asymmetry of the distribution. The closer the Characteristic Exponent α is to the upper limit of the permissible range, the less significance the skewness has in terms of shifting the shape of the distribution away from the corresponding symmetric distribution. At the limit $\alpha = 2.0$, the Normal distribution, the Skewness Parameter $\beta = 0$ becomes irrelevant as the distribution is symmetric.

The Scale Parameter γ lies in the open interval $(0, \infty)$, and is a measure of the spread of the distribution. If $\alpha = 2.0$, the Scale Parameter γ is directly proportional to the standard deviation: $\gamma = \sigma/\sqrt{2}$. However, the Scale Parameter γ is finite for all Levy-stable distributions, despite the fact that the standard deviation is infinite for all $\alpha < 2.0$. Thus, the Scale Parameter γ can be regarded as a generalization of the standard deviation, first reliably quantified by McCulloch (1986).

The Location Parameter δ is a rough measure of the midpoint of the distribution. A change in δ simply shifts the graph of the distribution left or right, hence the term “location.”

There are a number of parameterizations of stable laws (Nolan (1997, 1998, 2005, and 2020)). Two are predominant in financial applications. Nolan's S0 is useful in theoretical work as it is continuous in all four parameters; Nolan's S1 is often used because the location parameter is the mean. We used the S0 where δ is the mean where $\alpha > 1.0$, which is always the case in the data analyzed here. Also, S0 allows for the computation of confidence intervals essential to our work.

NCREIF Indicators: Capital Performance and Property Operations

Since its inception in 1978, the NCREIF Property Index (NPI) has been a measure of investment performance, a measure of returns in total and returns decomposed into income and capital subsets and disaggregated by various property characteristics and geographic locations. In a sense, the NPI is a measure of returns on a "portfolio" of institutionally-owned commercial real estate, which is why the headline returns reported are value-weighted. Fortunately, NCREIF has collected more information about the economic and operating performance of domestic commercial property since 2000 that can be extracted and reconstructed to create other data series.

Furthermore, unlike most other real estate data series, NCREIF financial data are audited, accounting-based data because the source is information provided by owners or managers who are themselves fiduciaries subject to the highest standard of care under the law (29 U.S. Code § 1101-1114).

The issue of accounting treatment of capital expenditures and its affect on the capital appreciation and income return components of the NPI has been addressed from time to time. The first published article on the subject, Young, Geltner, McIntosh and Poutasse (1995), focused on the performance formulas employed in the NPI. In that paper, the denominator was simplified to the periodic change in market value, and the capital expenditures were treated as cash flow expense rather than as a debit against market value as in the traditional NPI capital appreciation formula. While simple to compute, the interpretation of the return results have been questionable vis-à-vis other asset classes. The persistent problem has been the magnitude and categorization of capital expenditures. Fortunately, NCREIF accounting policies changed to breakout categories of capital expenditures to allow the curious to distinguish between commonplace, routine capital expenditures for maintenance of property, for making spaces suitable for tenants, and for leasing commissions and extraordinary capital expenditures associated with purchase, sale, or major renovation or expansions of space within properties.

In 2015, NCREIF released three refined data series called the Market Value Index (MVI), the Free Cash Flow Yield (FCFY), and the Capital Expense Ratio (CXR) collectively known by the formal name NCREIF Indicators: Capital Performance and Property Operations. The first two series are used in this paper for the simple reason that the sum of MVI and FCFY in any quarter is the quarterly total return for the property. The percentage amounts can be added to get the total return because both the MVI and the FCFY share the same denominator, namely the market value

at the beginning of the quarter. This simple denominator is also an important departure from the more complicated one employed in the classic NPI.

In accounting terms, the MVI is the unrealized gain from an appraisal-based change in value rather than from the actual sale of the asset and the FCFY is the net cash flow that could be distributed to the property owner. Together these statistics represent the capital and net cash income gains on a property in a quarter. At first glance, these terms, capital and income gains, might appear similar to Price and Cash Flow Indices that NCREIF has provided researchers and practitioners in detailed spreadsheets or in on-line query tools.

However, comparing the classic NPI Price and Cash Flow formulas with the MVI and FCFY formulas will demonstrate that these various measures are indeed quite different. It would not be an exaggeration to say that the Price and Cash Flow formulas are, at best, improperly named and often misused owing to the false impression their names denote rather than connote.⁴

The NPI formula is:

$$R_t = \frac{MV_t - MV_{t-1} + PS_t + NOI_t - CI_t}{MV_{t-1} - (PS_t/2) - (NOI_t/3) + (CI_t/2)} \quad (2)$$

where R_t is the Total Return for period t , MV_t is the Market Value at the end of period t , MV_{t-1} is the Market Value at the beginning of period t , PS_t is any Partial Sales in period t , NOI_t is the Net Operating Income in period t , and CI_t is the Capital Expenditures (Improvements) in period t .

The Price (P_t) change for a quarter is:

$$P_t = \frac{MV_t - MV_{t-1} + PS_t}{MV_{t-1} - (PS_t/2) - (NOI_t/3) + (CI_t/2)} \quad (3)$$

The Cash Flow (CF_t) for a quarter is:

$$CF_t = \frac{NOI_t - CI_t}{MV_{t-1} - (PS_t/2) - (NOI_t/3) + (CI_t/2)} \quad (4)$$

Significantly, within the classic NPI computational formula, there are two issues related to the calculation of the so-called Price and Cash Flow Indices. First, the same denominator is used as in the NPI. This denominator is based on the Modified Dietz Method (Dietz, 1966) and designed to provide an estimate of an Internal Rate of Return (IRR) for a *single quarter assuming monthly cash flows*. However, as constructed, the Price Index and Cash Flow Index *were not* Real Estate Return Distributions with New NCREIF Data Series

intended to be returns. Second, the ending value of the Price Index will be higher by all capital expenditures including capital spent for expansion and major renovations not just ordinary, routine capital expenses generally associated with tenant leases such as commissions and modifications to or construction of new tenant spaces. Notably, a property undergoing expansion or major renovation is not the same property at the end of its accounting period as it was before the renovation or expansion. In other words, the Price Index is not a “same store” analysis of price changes; the store has changed substantially physically, functionally, and economically in the quarter as a result of the owner’s strategic or business decision.

The Market Value Index (MVI) is designed to deal with these issues in two ways. First, the denominator of the MVI formula is simply the beginning market value for the quarter: MV_{t-1} . This treatment is more consistent with the way price indices are calculated for other asset classes. Second, the MVI excludes properties in quarters where there is major renovation or expansion. These exclusions eliminate computations in quarters where a property is likely changing its physical, functional, or financial economic character.

Thus, the MVI is an equally weighted average of quarterly changes in reported market value pure and simple with no major capital expenditures involved. It is a refinement of the NCREIF Query Tool’s “Price Index” in that it is generated strictly from period-to-period changes in the reported market value for each property rather than from changes in the Capital Return including Capital Expenditures, which is a return measure inaccurately described as a change in price.

The Free Cash Flow Yield (FCFY) is a measure of the amount of a property’s quarterly operating net cash flow available for distribution to investors/owners expressed as a fraction of market value. This is a *periodic measure that is not indexed*, that, like the MVI, excludes properties with major capital expenditures for expansion or renovation when such expenditures exceed the filter rule described below, and that more accurately reflects the actual net cash that could be distributed to investors or to property owners in a quarter. This measure is similar to the concept of “free cash flow” used in the finance literature and in corporate financial reporting.

The FCFY is a periodic measure of the cash that investors can expect from operation of commercial property. In some sense, it is the cash in hand each quarter after all operating and everyday capital expenses have been paid. To many investors, the cash available for distribution is one of the principal reasons for investing in commercial real estate, an asset that has a relatively stable net cash flow stream due to the terms of leases that underpin a property’s economic performance. Stability and reliability of this net cash flow stream is important to a wide range of institutional investors who must have cash available for distribution to plan beneficiaries. With the introduction of the FCFY, those investors have, for the first time, a way of assessing the history of net cash flow available for distribution from the real estate asset class.

In summary, the new measures introduced in the NCREIF Indicators: Capital Performance and Property Operations quarterly data sets differ from the NPI in several ways:

1. Because the new measures that sum to the quarterly total return, namely MVI and FCFY, are not intended as estimates of a quarterly IRR, no adjustment has to be made to the denominator of the NPI's formula for estimated cash flows within the quarter. The denominator is simply the Beginning Market Value.
2. Unlike the NPI Income Return component based on NOI, the FCFY is calculated by subtracting ordinary, predominantly lease-related Capital Expenditures, from the NOI to get a measure of net cash flow.
3. The MVI, unlike the NPI Appreciation Return component where all Capital Expenditures are deducted from the Ending Market Value, there is no deduction for Capital Expenditures. Thus, the MVI simply measures the change in Market Value plus Partial Sales, if applicable.
4. While some Capital Expenditures are necessary to maintain a constant utility of a property, a price or value index should not reflect an increase in the value of the property due to a major expansion or renovation that involves new capital investment. Similarly, the FCFY measure should not have major, extraordinary Capital Expenditures deducted. Thus, for both the MVI and the FCFY, a property is excluded from the calculation during any quarter where expansion or major renovations are taking place.
5. The NPI returns have always been value-weighted as they represent the universe of properties reported to NCREIF. In a sense, the NPI returns are the returns of a particular "portfolio" of institutionally-owned commercial real estate. The new measures are equal-weighted to reflect data representative of the universe of domestic commercial real estate regardless of ownership. When dealing with sample statistics from a universe of domestic commercial real estate, equal-weighted statistics are preferable.

MVI and FCFY Defined

MVI is computed for each qualified property as the sum of Ending Market Value and Partial Sales⁵ divided by Beginning Market Value minus 1.0 for each quarter. To deal with the "same store" issue discussed above, in any quarter where the absolute value⁶ of specified capital expenditures exceeds a fraction of Beginning Market Value the property's MVI computation is disqualified or excluded from the data series.

The MVI formula is:

$$\left[\frac{(MV_t + PS_t)}{MV_{t-1}} \right] - 1 \quad (5)$$

or alternatively

$$\frac{(MV_t - MV_{t-1} + PS_t)}{MV_{t-1}} \quad (6)$$

where MV_t is Market Value and PS_t is Partial Sales reported to NCREIF in quarter t and MV_{t-1} is the Market Value at the end of the prior quarter, in other words, at the beginning of the current quarter.

FCFY is computed for each property as the quantity Net Operating Income minus Capital Improvements divided by Beginning Market Value for each quarter. Notice that major capital expenditures for expansions or renovations are not included in the formula, while ordinary, frequently occurring capital expenses related to leasing commissions, tenant improvements, and other routine building maintenance are included in the term CI_t . Accordingly, in quarters where the absolute value of capital improvements defined in the filter rule exceed a fraction of Beginning Market Value, the property's FCFY computation is excluded from the data series.

Thus, the FCFY formula is:

$$(NOI_t - CI_t) / MV_{t-1} \quad (7)$$

where NOI_t is the Net Operating Income, CI_t is Capital Improvements related to tenant expenses or retention and routine capital expenditures, and MV_{t-1} is Market Value reported to NCREIF at the end of the prior quarter.

As mentioned above, MVI_t and $FCFY_t$ are components of total return for a quarter when substantial capital expenditures have not occurred. Expressed in algebraic form, the total return for a property in a quarter would be simply:

$$TR_t = MVI_t + FCFY_t \quad (8)$$

The Data Series Filter Rule

One of the essential differences between the classic NCREIF NPI returns series and the new NCREIF Indicators: Capital Performance and Property Operations data series is the filter rule that excludes particular NPI properties. How this filter rule was crafted is so consequential to an understanding of the new data series that we believe that an extended quotation from Young, Fisher, and D'Alessandro (2017) is warranted.

To ensure that, within reasonable bounds, a particular property retains its physical continuity throughout a quarter, there must be a way to identify properties that have not had substantial, material changes to the physical asset within the quarter. If the changes are substantial, the property should be excluded for that quarter or for subsequent quarters until such time as the property becomes stable physically, functionally, or economically, i.e., when the property returns to a state of constant utility.

Prior to 2000, only total Capital Improvements were reported to NCREIF. Subsequently, additional subcategories of capital improvements gave us more information on the composition of total Capital Improvements. In particular, the subcategories included Additional Acquisitions Costs, Leasing Commissions, Tenant Improvements, Building Improvements, Building Expansion, and Other Capital Improvements.

We divide these subcategories into two groups: those that are typical recurring capital expenses related to changing tenancy and ordinary repairs, and those that are occasional, high-dollar-value capital expenditures that alter the physical, functional, or economic condition of a property. Leasing Commissions, Tenant Improvements, and Building Improvements fall into the former group and are included in Capital Expenses in the FCFY ... series. Additional Acquisitions Costs, Building Expansion, and Other Capital Improvements fall into the latter group and are all candidates for filtering properties for exclusion within all three series.

We can use the detailed data on capital expenditures from the post-2000 era to create filter rules for excluding properties undergoing substantial capital expenditures prior to 2000. We are not able to say with certainty that properties filtered will be 100% accurately identified. We must strike a reasonable balance based on indicators we find in the existing data and judgments about the reasonableness of the filter ratio.

Thus, we have chosen to filter only those subcategories of capital expenditures in the subcategories of Additional Acquisitions Costs, Building Expansion, and Other Capital Improvements that show an absolute value greater than 5% of Beginning Market Value of a property in any quarter. First, we compute for each property type the fraction of post-2000 observations that are filtered and then use that fraction to establish a filter rule for the pre-2000 era where we have only one statistic for total Capital Improvements.

We tried several filter rules for pre-2000 data and found that an absolute value of total Capital Improvements greater than 10% of Beginning Market Value provided the most similar fraction of excluded quarters for most property types and for the aggregate of all properties in the NPI. Exhibit 1 [*not included in this quotation*] shows the observations and fractions of properties excluded in both the pre-2000 and post-2000 eras. In the post-2000 era, 2,079 of the 289,543 quarterly observations for all properties in the NPI were filtered (satisfied the rule for exclusion from the data set), a total of 0.72% of all observations. Interestingly, in the pre-2000 era, 958 of the 132,635 quarterly observations for all properties in the NPI were filtered, also a total of 0.72% of all observations despite some differences in percentages pre- and post-2000 when disaggregated by individual property type.

The net effect of this filter rule upon the number of annual observations in the NPI data set relative to the Capital Performance and Property Operations data set was to reduce the sample size of the set used in this paper by about 2% from the NPI data set.

Analytical Tools for Levy-stable Distributions

A variety of commercially-available computer software products contain statistical routines including Mathematica, MATLAB, Maple, SAS, and SPSS; an open-source, free application known as R; as well as spreadsheet applications like Microsoft's Excel and Apple's Numbers. Statistical routines for the analysis of Levy-stable distributions are less commonplace.

One notable exception is Wolfram Research's Mathematica, a symbolic-logic computing software application that offers an extensive suite of statistical tools including Levy-stable distributions. Independent applications written in different source code such as Fortran can be integrated with standard Mathematica tools via a sub-routine know as Mathlink. John Nolan's STABLE for Mathematica, DOS-based Fortran application that uses Mathlink is referenced in his 2020 book and available at these web sites:

<http://www.RobustAnalysis.com>

<http://edspace.american.edu/jpnolan>

The standard error and confidence interval results of this paper were computed with Nolan's STABLE application.

Tests and Results

The analytical toolbox available to most researchers and certainly most students and practitioners includes tools specifically designed for data sets presumed to be normally distributed, so it is helpful to start with the ordinary parametric statistics of mean, median, standard deviation (or variance), skewness, and kurtosis. If the sample was Normal, the mean and the median would be identical, the skewness would be 0.0, and the kurtosis would be 3.0. Exhibit 1 shows statistics for the entire data set of 135,655 samples over the period 1990 to 2021 as well as the summary statistics for each of the four property types. In not a single instance, save one Apartment property series in 1990 where test is indeterminate, do the statistics agree with those of the Normal Distribution.

If the analytical toolbox contains the familiar Jarque-Bera test for the likelihood that a sample distribution is Normal. The 95% confidence level χ^2 test statistic for 135,655 samples Normally distributed Normally would be 1.552. However, the χ^2 test statistic for the distribution of the actual 135,655 samples of NCREIF property returns presented here is 17,319,778. Thus, the Jarque-Bera test of these data shows an infinitesimally small probability that the actual distribution is Normal. But, we have digressed.

Using Mathematica's Maximum Likelihood Estimation (MLE) routine, we fit a Levy-stable distribution to each set of annual returns arranged by calendar year and property type over the

calendar years 1990 to 2021. To test whether the parameters varied during the sample period, Levy-stable parameters were estimated for sets composed of the residuals aggregated across calendar years and property types. These results are tabulated in Exhibit 5 and are displayed graphically together with 99% confidence intervals (for all years where standard errors could be ascertained) in Exhibit 4 for the parameter α , β and γ . Summary statistics for All Properties and the four property-type subsets are shown in Exhibit 2.

Despite differences in estimated α in the MVI+FCFY series versus α reported in earlier NPI-based studies, the Characteristic Exponents α still evidence statistically significant departures from Normality with 99% confidence. Covering the study period 1990 to 2021, the estimated Characteristic Exponent α for All Properties in the MVI+FCFY series is 1.398. Corresponding estimates by property types are 1.443 for Office properties, 1.432 for Retail properties, 1.351 for Industrial properties, and 1.455 for Apartment properties.

Over the same period, the return distributions are negatively skewed (Skewness Parameter β) for Office and Retail properties and positively skewed for Industrial and Apartment properties. This result is directionally identical to the results reported in Young and Brown (2012), except for Industrial that has gone from negative to positive.

With respect to the Scale Parameter γ , a generalization of the standard deviation or, more commonly, the risk of an asset, the figures tabulated in Exhibit 2 are somewhat greater than the corresponding figures in Young and Brown (2012) except for Office properties that are lower. No doubt, the filter rule employed in the construction of the MVI +FCFY series is a contributor to these differences. The Scale Parameter γ for All Properties is shown as 0.069 with ranges from a high of 0.074 for Office properties and a low of 0.060 for Apartment properties.

Given the popular anthropomorphic description of leptokurtic distributions relative to Normal distributions, namely leptokurtic distributions have fat tails (okay, that is not really anthropomorphic), weak shoulders, and tall bodies with pointed heads, it may be of interest to note that a reasonably close approximation of the Scale Parameter γ can be derived from the Semi-Interquartile Range easily obtained from ordinary parametric statistics of real estate return distributions. Exhibit 3 shows dispersion estimates of the MVI+FCFY series for Standard Deviation (Second Moment) of the Normal Distribution, Semi-Interquartile Range, and the Scale Parameter γ . For All Properties, the Semi-Interquartile range is 0.062 and the Scale Parameter γ is 0.069 for a difference of only -0.007 . Similarly, small differences are found across all four property-type disaggregations.

In the case of Characteristic Exponents α_t estimated by calendar year and property type, 100% of the samples tabulated in Exhibit 2 (for which standard errors could be computed) by property type were distinct statistically from 2.0, the Characteristic Exponent of the Normal distribution, with 99% confidence. In the case of residuals aggregated across property type, all sample Characteristic Exponents α_t were distinct from 2.0 with 99% confidence.

From Exhibits 2 and 5, for the entire 1990 to 2021 sample period, estimates of Characteristic Exponents together with their 99% confidence interval ranges are 1.398 ± 0.010 for all four property types combined, 1.443 ± 0.022 for Office properties, 1.432 ± 0.026 for Retail properties, 1.351 ± 0.016 for Industrial properties, and 1.455 ± 0.023 for Apartment properties. In all instances, these Characteristic Exponent estimates are lower and farther removed from the Normal Distribution, than reported in any previous study of NCREIF NPI data.

Exhibit 5 displays the sample Characteristic Exponents α_t of both the aggregated and individual property type residuals. It appears that α_t could be time-invariant within property types. Furthermore, Exhibit 4 that shows graphical representations of these data suggests that α_t likely varies across property type and somewhat more so than in any previous study of NCREIF NPI data.

The graphical Exhibit 4 shows the Characteristic Exponent for each property type and the aggregate over the full 1990 to 2021 time period along with the 99% confidence bands. In the case of the Characteristic Exponent, Office, Retail, and Apartment property types are statistically indistinguishable from one another while Industrial stands apart. These differences among property types deviate from conclusions of prior studies using non-MLE analytical techniques where more statistical similarities were observed.

The above analysis implies that over the sample period 1990 to 2021 (1) real estate investment risk was heteroscedastic for properties of a type and in the aggregate; (2) during virtually all sample subperiods and across property type, Levy-stable infinite-variance skewed asset-specific risk functions with a Characteristic Exponent α of approximately 1.398 with a 99% confidence interval range of ± 0.010 modeled the observed distributions of return residuals better than Normally distributed risk candidates; and (3) property type differences in the Characteristic Exponent across property types are likely, which begs the question for further research into other dimensions along which distinct differences may emerge and investment strategies or tactics that might be employed to take advantage of these differences. Lease duration and desirability of property types as investments in changing economic conditions appear to be a good candidates for further research

Conclusions

Once committed to investing in commercial real estate, return on investment comes in two forms: capital appreciation and periodic net cash flow.⁷ MVI and FCFY are superior measures of these components rather than reformulations of the classic NPI that have been used for decades.

The MLE analysis of this study supports the conclusion that there is no single value for the Characteristic Exponent of asset-specific risk across property type. Nonetheless, all four NCREIF property types exhibit statistically significant departures from the Normal distribution at the 99% confidence level for all thirty-two years of this study.

As a practical matter, few, if any, investors have the financial resources to acquire commercial real estate portfolios even of a single property type to approach the risk reduction magnitudes available in other liquid, securitized asset classes like common stocks such that the non-systematic and systematic risks of the portfolio are comparable.

Once invested in real estate, the systematic risk is a fact of ownership. However, investors may affect for good or for bad the non-systematic, property-specific risk. Arguably, this is more so in the real estate asset class than in other investments where ownership and control are separated (albeit private equity is more akin to real estate in that its performance can be managed or directly impacted by the owner).⁸

Risk and reward are said to be positively related. But, risk, positive or negative, can be managed. Reward is just accounting for the degree to which risk has been realized, either positive or negative, on-the-ground ex post.

The analysis of this article implies that over the sample period 1990 to 2021 (1) real estate investment risk was heteroscedastic for properties of a type and in the aggregate; (2) during virtually all sample subperiods and across property type, Levy-stable infinite-variance skewed asset-specific risk functions with a Characteristic Exponent α of approximately 1.398 with a 99% confidence interval range of ± 0.010 modeled the observed distributions of return residuals better than Normally distributed risk candidates; and (3) property type differences in the Characteristic Exponent across property types are likely, which begs the question for further research into other dimensions along which distinct differences may emerge and investment strategies or tactics that might be employed to take advantage of these differences.

The conclusions of this study reinforce the earlier result that for institutional-grade real estate portfolios, the appropriate degree of risk reduction across multiple risk factors (locational, economic, etc.) could only be achieved by purchasing most of the institutional-grade properties in the U.S.—an obvious practical impossibility. This implies that institutional real estate portfolio management must be concerned with the asset-specific risk component of each property included in the portfolio with perhaps lesser consideration given to market/systematic and market-sector risk components. In street parlance, this means that your real estate eggs are always in one basket, so it behoves you to watch those individual eggs very carefully. This elevates in importance the otherwise pedestrian tasks of on-the-ground management and entrepreneurship.

In this study, property type differences in performance matter, more so than demonstrated in previous studies.

The fundamental reasons for differences in property-type performance boil down to the sources of net revenue and the market forces impacting the properties. In essence, lease structure and obsolescence or technological change matter most. These are subjects worthy of future research on specific property types irrespective of return differences that may or may not exist

across property types. NCREIF Indicators: Capital Performance and Property Operations data sets offer ample statistics to pursue this research.

Slowly, knowledge of the non-Normal characteristics of real estate return distributions has spawned inquiry into processes and applications beyond the well worn mean-variance models. For example, Brown (2004) notes differences in skewness of returns between institutional-grade commercial property and direct, private investment property where investors can influence the outcome. Commercial property returns, subject to bond-like leases for a share of their total value estimates, often exhibit bond-like negative skewness. By contrast, Brown finds that direct property returns are more likely to be positively skewed leading to the conjecture that the value-added activities of owners is, on balance, a net positive for performance.

Brown and Young (2011) offer a middle-ground solution to measuring downside risk in real estate by means of so-called Coherent Risk Measures, an improvement both quantitatively and qualitatively over the more familiar Value At Risk (VaR) measures used in banking institutions.

Mathematica and tools built upon the platform collectively called Wolfram Demonstrations Project offer quick, easy, and graphically interesting ways to probe aspects of return distributions and examine their consequences. For example, Brown's "Forming the Efficient Frontier When Returns are Non-Normal" demonstration shows efficient frontiers generated Normal versus Levy-stable distribution under user-selectable variations of Levy-stable distribution parameters.⁹

Applications of Levy-stable distribution parameters to real estate return distributions to date have generally involved proprietary or difficult-to-acquire databases. NCREIF data from the new MVI and FCFY series are available to members and academics pursuant to a "disaggregation request." Investment Property Databank (IPD) data required processing by an employee of IPD but the data in raw form were unavailable to the other researchers on the study. These restrictive procedures encumber would-be researchers and doubtless limit the investigations that might afford more and varied insights into the nature and implications of real estate return distributions. We can hope that the efforts of those who have been able to penetrate the labyrinth thus far will convince database gatekeepers that openness and transparency can benefit the entire industry.

Lastly, it may be time to re-think the entire enterprise of trying to specify the shape of the distribution in order to derive two or four parameters. The on-going revolution in Big Data, machine learning, click-throughs and apps have erased the stigma previously associated with data mining. Maximum entropy theory, which asserts that the probability distribution that most likely describes the current state of knowledge is the one with the largest entropy, is perhaps the most used optimization technique in data analysis today as discrete Bayesian approaches replace MLE and continuous frequentist methods.

If real estate exists at the intersection of law and economics, its data span alphabetic and numeric. This means that Natural Language Processing may soon play a role as big as spreadsheets do now. As skilled hackers in a foreign country can liberate data overnight in a

variety of ways from any number of sources, one wonders when incarcerating data will go the way of locking Pilgrims in stocks. What we have today is science being turned on its head: data determines theory, not the other way around. Real estate has long suffered from a reputation of having hundreds of years of history unimpeded by progress. That needs to change and, we believe, research in the real estate asset class would benefit from progress in this direction.

Lastly, groups make decisions using the Probability Distribution Function (PDF) as their interest is in finding the mean. Individuals, however, make decisions using the Cumulative Distribution Function (CDF) largely because their decisions involve fewer choices than those available to groups and because individuals are trying to avoid being hit by a train. While we have shown that forming portfolios of investable real estate assets to reduce risk in accordance with the dominant paradigm is virtually impossible, that does not mean that portfolios are not formed. Our data is a sample portfolio formed by society. Generally speaking, every property has an asset manager, one or more property managers, more than ten tenants, more than one hundred employees, more than a thousand customers, each with multiple family members. Our data represents that multitude, each individually making bets about their personal economic survival.

The inverse of the CDF is the quantile. Below are a number of quantiles, essentially “bins” into which individuals self-select based on their particular risk tolerance. When those individuals make faulty assumptions about the probabilities of success or failure, they land in a different bin than the one they aimed for. The right column below records how much they missed their target total return in the 1990 to 2021 period if the World of commercial real estate is as risky as our data indicates.

Annual Total Returns by Quantile

Quantile	Distribution Stable $\alpha = 1.4$	Distribution Normal $\alpha = 2.0$	Difference
10th	-14.9%	-8.8%	-6.1%
25th	-6.7%	-4.7%	-2.1%
35th	-3.7%	-2.7%	-1.1%
50th	0.0%	0.0%	0.0%
65th	3.7%	2.7%	1.1%
75th	6.7%	4.7%	2.1%
90th	14.9%	8.8%	6.1%

This paper has focused on investment returns for institutional-grade real property merely as a proxy for a society that needs and wants more precise risk measures.

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Exhibit 1
Normal Distribution Statistics for NCREIF MVI+FCFY Series
for All Properties, 1990 to 2021

Year or Period	Mean	Median	Std Dev	Skewness	Kurtosis	Number of Obs.
2021	26.2	22.0	24.4	1.4	4.7	8,732
2020	5.4	5.3	10.9	0.4	9.6	7,965
2019	8.8	8.1	10.2	0.9	20.8	7,067
2018	9.6	8.6	10.1	2.2	42.5	6,947
2017	8.9	7.9	9.6	1.1	11.4	6,480
2016	9.3	8.8	10.4	1.5	17.2	6,472
2015	13.4	12.4	12.7	2.6	47.1	6,010
2014	11.9	11.0	12.0	4.0	85.0	6,094
2013	10.7	10.2	11.2	0.7	10.2	6,145
2012	10.0	9.8	12.9	2.4	43.9	6,239
2011	12.8	12.4	16.5	16.8	752.6	5,499
2010	10.7	10.5	16.5	3.8	98.5	5,375
2009	-15.7	-15.1	13.5	0.6	12.8	5,605
2008	-5.4	-5.1	14.4	6.5	177.9	5,476
2007	12.9	11.5	11.4	1.5	11.2	4,747
2006	15.7	13.8	13.7	2.2	24.0	4,094
2005	18.3	16.2	15.6	1.2	7.2	3,511
2004	12.5	11.9	13.2	0.9	9.6	3,433
2003	8.1	8.9	11.8	-0.4	5.7	3,291
2002	6.5	7.9	11.2	-0.3	5.0	3,036
2001	7.8	8.7	10.6	0.4	17.8	2,662
2000	12.2	10.9	11.5	1.5	16.5	2,318
1999	11.7	10.7	9.5	1.1	8.1	2,066
1998	15.7	13.1	12.4	1.8	8.4	1,938
1997	15.3	12.5	14.2	1.9	9.6	1,937
1996	11.8	10.7	13.0	1.6	21.4	1,976
1995	10.2	10.2	12.3	0.1	6.4	1,722
1994	8.4	9.2	12.8	-0.1	4.8	1,702
1993	2.9	5.7	14.6	-0.3	5.2	1,837
1992	-2.6	0.3	15.2	0.3	17.2	1,889
1991	-4.6	-1.8	14.3	-1.0	1.7	1,805
1990	0.8	4.0	13.1	-1.3	3.4	1,585
1990-21	9.4%	9.1%	15.9%	2.1	55.2	135,655

Normal Distribution Statistics by Property Type, 1990 to 2021

Type	Mean	Median	Std Dev	Skewness	Kurtosis	No. of Obs.
Office	6.3%	7.1%	16.7%	3.9	147.9	30,155
Retail	7.3%	8.0%	14.1%	2.9	85.8	22,294
Industrial	12.1%	11.0%	17.2%	1.4	10.6	54,331
Apartment	9.0%	8.4%	12.5%	0.4	5.3	28,875

Exhibit 2
MLE Levy-stable Distribution Parameters
for NCREIF MVI+FCFY Series by Property Type
Total Annual Return Residuals & Mean Returns & Number of Observations
1990 to 2021

Property Type	α	β	γ	Mean Return	Number of Observations
Office	1.443 **	-0.230	0.074	0.063	30,155
Retail	1.432 **	-0.233	0.061	0.073	22,294
Industrial	1.351 **	0.096	0.071	0.121	54,331
Apartment	1.455 **	0.093	0.060	0.090	28,875
All Properties	1.398 **	-0.000	0.069	0.094	135,655

Statistically significant confidence of non-Normality ($\alpha \neq 2.0$): ** = 99% confidence

α is the Characteristic Exponent, and only equals 2.0 for the Normal distribution

β is the Skewness Parameter in the range -1.0 to +1.0

γ is the (positive) Scale Parameter which measures the spread of the distribution about δ

Note: The mean returns are shown in Exhibit 3 simply for purposes of completeness.

Exhibit 3
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Scale Parameter γ
1990 to 2021

Property Type	Std Dev	Semi-IQR	γ	Semi-IQR minus γ	Std Dev minus γ
Office	0.167	0.068	0.074	-0.006	0.093
Retail	0.112	0.056	0.061	-0.005	0.051
Industrial	0.172	0.066	0.071	-0.005	0.101
Apartment	0.125	0.055	0.060	-0.005	0.065
All Properties	0.159	0.062	0.069	-0.007	0.090

Exhibit 4
 MLE Stable Distribution Parameters for NCREIF MVI+FCFY Series by Property Type
 1990 to 2021
 (bands indicate 99% confidence interval)

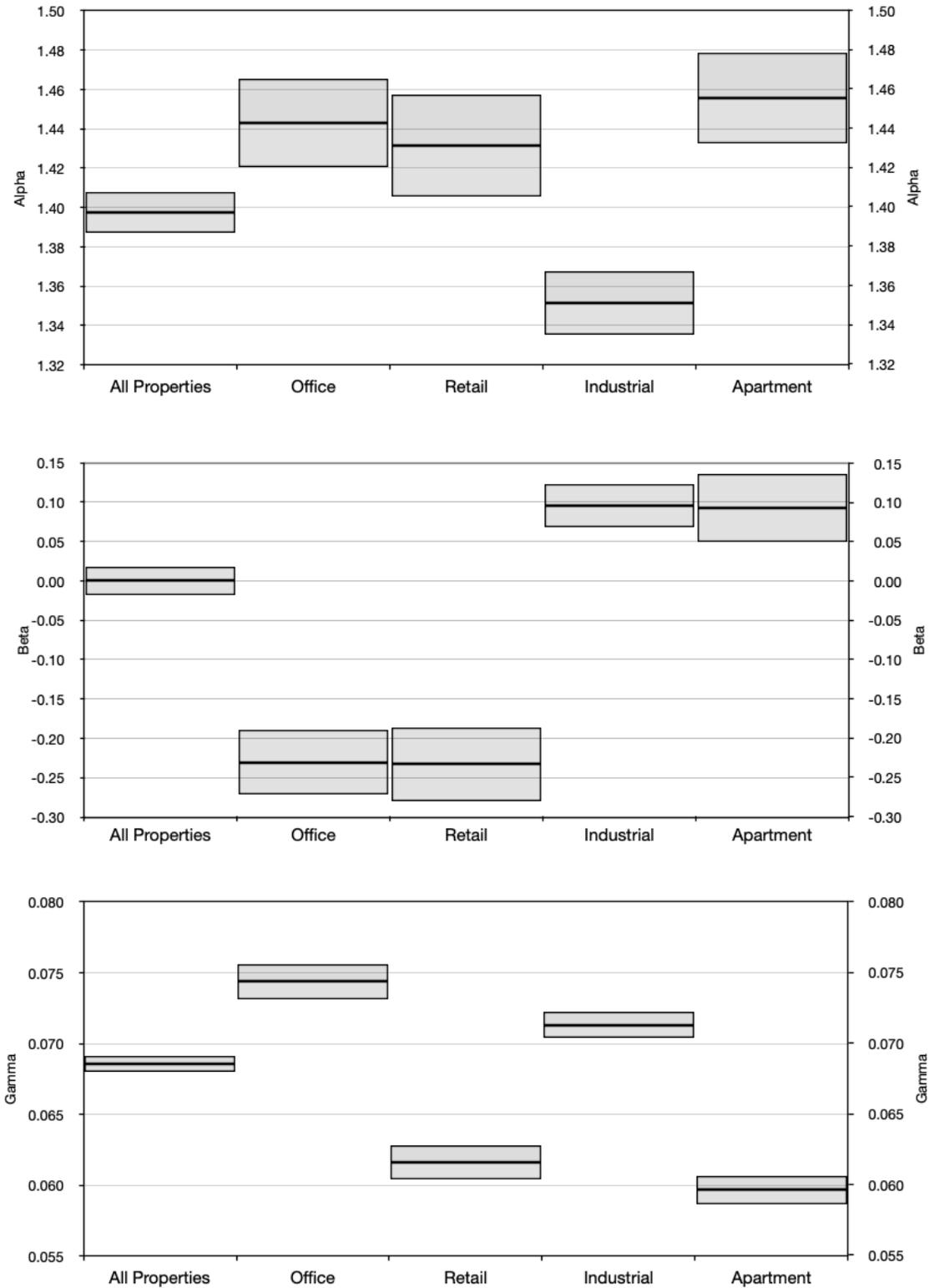


Exhibit 5
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Total Annual Return Residuals & Mean Returns & Number of Observations

All Properties Combined:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2021	1.554 **	0.928	0.129	0.262	8,732
2020	1.646 **	0.073	0.057	0.054	7,965
2019	1.487 **	0.226	0.047	0.088	7,067
2018	1.508 **	0.354	0.048	0.096	6,947
2017	1.423 **	0.339	0.043	0.089	6,480
2016	1.484 **	0.111	0.045	0.093	6,472
2015	1.501 **	0.371	0.056	0.134	6,010
2014	1.518 **	0.266	0.051	0.119	6,094
2013	1.487 **	0.136	0.051	0.107	6,145
2012	1.517 **	-0.000	0.057	0.100	6,239
2011	1.683 **	0.000	0.071	0.128	5,499
2010	1.708 **	-0.093	0.087	0.107	5,375
2009	1.822 **	-0.545	0.082	-0.157	5,605
2008	1.780 **	-0.309	0.076	-0.054	5,476
2007	1.491 **	0.535	0.053	0.129	4,747
2006	1.471 **	0.563	0.053	0.157	4,094
2005	1.520 **	0.578	0.075	0.183	3,511
2004	1.634 **	-0.000	0.069	0.125	3,433
2003	1.571 **	-0.319	0.060	0.081	3,291
2002	1.481 **	-0.465	0.054	0.065	3,036
2001	1.402 **	-0.312	0.044	0.078	2,662
2000	1.311 **	0.284	0.042	0.122	2,318
1999	1.406 **	0.278	0.041	0.117	2,066
1998	1.334 **	0.617	0.049	0.157	1,938
1997	1.343 **	0.576	0.056	0.153	1,937
1996	1.342 **	0.208	0.050	0.117	1,976
1995	1.385 **	-0.087	0.053	0.102	1,722
1994	1.419 **	-0.161	0.059	0.084	1,702
1993	1.373 **	-0.539	0.067	0.029	1,837
1992	1.481 #	-0.913	0.075	-0.026	1,889
1991	1.528 #	-0.915	0.078	-0.046	1,805
1990	1.339 **	-0.784	0.060	-0.002	1,585
1990-21	1.398 **	-0.000	0.069	0.094	135,655
99% conf.	0.010				

Exhibit 5 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Total Annual Return Residuals & Mean Returns & Number of Observations

Office Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2021	1.458 **	0.257	0.054	0.074	1,509
2020	1.524 **	-0.320	0.047	0.019	1,402
2019	1.453 **	0.070	0.044	0.066	1,250
2018	1.483 **	-0.070	0.042	0.068	1,295
2017	1.369 **	-0.114	0.037	0.064	1,247
2016	1.478 **	-0.248	0.046	0.067	1,232
2015	1.369 **	0.132	0.058	0.113	1,190
2014	1.499 **	0.000	0.059	0.098	1,219
2013	1.472 **	-0.000	0.055	0.086	1,236
2012	1.488 **	-0.076	0.056	0.079	1,263
2011	1.563 **	0.170	0.072	0.115	1,239
2010	1.648 **	-0.000	0.085	0.080	1,274
2009	1.886 **	-0.317	0.095	-0.181	1,397
2008	1.788 **	-0.411	0.083	-0.064	1,371
2007	1.511 **	0.558	0.071	0.157	1,106
2006	1.459 **	0.522	0.072	0.163	1,012
2005	1.460 **	0.449	0.079	0.178	963
2004	1.610 **	-0.149	0.072	0.096	978
2003	1.505 **	-0.604	0.063	0.041	994
2002	1.521 **	-0.750	0.064	0.020	921
2001	1.519 **	-0.448	0.056	0.054	832
2000	1.265 **	0.388	0.048	0.131	673
1999	1.565 **	0.475	0.049	0.121	589
1998	1.341 **	0.834	0.064	0.197	515
1997	1.555 **	0.759	0.083	0.211	425
1996	1.646 **	0.543	0.073	0.146	450
1995	1.514 **	0.000	0.075	0.090	379
1994	1.566 **	-0.305	0.087	0.062	412
1993	1.645 **	-0.738	0.091	-0.016	463
1992	1.685 #	-0.934	0.097	-0.090	442
1991	1.744 #	-1.000	0.103	-0.123	448
1990	1.514 #	-0.938	0.077	-0.058	411
1990-21	1.443 **	-0.230	0.074	0.063	30,155
99% conf.	0.022				

Exhibit 5 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Total Annual Return Residuals & Mean Returns & Number of Observations

Retail Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2021	1.310 **	-0.088	0.043	0.072	1,213
2020	1.475 **	-0.609	0.052	-0.028	1,170
2019	1.246 **	-0.453	0.038	0.030	1,123
2018	1.258 **	-0.484	0.033	0.043	1,082
2017	1.284 **	-0.224	0.029	0.060	1,072
2016	1.413 **	-0.000	0.035	0.083	1,035
2015	1.454 **	0.460	0.050	0.137	979
2014	1.443 **	0.355	0.044	0.123	1,007
2013	1.300 **	0.268	0.046	0.117	993
2012	1.411 **	0.070	0.070	0.103	1,027
2011	1.677 **	0.197	0.072	0.126	974
2010	1.585 **	-0.297	0.074	0.099	892
2009	1.765 **	-0.518	0.072	-0.126	819
2008	1.744 **	-0.421	0.073	-0.049	865
2007	1.367 **	0.537	0.040	0.116	719
2006	1.485 **	0.659	0.042	0.128	568
2005	1.513 **	0.682	0.065	0.193	471
2004	1.545 **	0.477	0.064	0.196	486
2003	1.505 **	0.106	0.050	0.147	440
2002	1.370 **	0.159	0.039	0.116	446
2001	1.216 **	-0.349	0.033	0.082	456
2000	1.239 **	-0.149	0.036	0.097	450
1999	1.412 **	0.154	0.041	0.115	422
1998	1.345 **	0.189	0.043	0.130	425
1997	1.186 **	0.000	0.043	0.107	465
1996	1.089 **	-0.262	0.040	0.071	513
1995	1.197 **	-0.567	0.046	0.052	380
1994	1.216 **	-0.291	0.040	0.066	362
1993	1.254 **	-0.464	0.049	0.044	408
1992	1.462 #	-0.924	0.059	0.002	378
1991	1.637 **	-0.731	0.066	-0.013	377
1990	1.293 **	-0.533	0.038	0.054	277
1990-21	1.432 **	-0.233	0.061	0.073	22,294
99% conf.	0.026				

Exhibit 5 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Total Annual Return Residuals & Mean Returns & Number of Observations

Industrial Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2021	1.554 **	0.949	0.123	0.409	4,188
2020	1.530 #	0.379	0.046	0.111	3,554
2019	1.578 **	0.551	0.048	0.131	3,151
2018	1.561 **	0.561	0.049	0.139	3,158
2017	1.517 **	0.449	0.053	0.123	2,748
2016	1.472 **	0.288	0.048	0.118	2,823
2015	1.514 **	0.419	0.058	0.147	2,525
2014	1.529 **	0.261	0.055	0.134	2,563
2013	1.554 **	0.092	0.058	0.115	2,617
2012	1.565 **	-0.000	0.065	0.101	2,608
2011	1.667 **	0.094	0.073	0.119	1,964
2010	1.717 **	-0.217	0.082	0.083	1,959
2009	1.795 **	-0.554	0.083	-0.163	2,076
2008	1.692 **	-0.300	0.069	-0.049	1,925
2007	1.622 **	0.403	0.051	0.129	1,831
2006	1.580 **	0.496	0.065	0.162	1,692
2005	1.719 **	0.323	0.081	0.182	1,357
2004	1.650 **	-0.249	0.070	0.122	1,264
2003	1.574 **	-0.326	0.059	0.086	1,152
2002	1.400 **	-0.542	0.048	0.072	984
2001	1.300 **	-0.174	0.038	0.093	767
2000	1.235 **	0.403	0.039	0.136	637
1999	1.214 **	0.109	0.037	0.120	592
1998	1.368 **	0.670	0.049	0.161	583
1997	1.439 **	0.775	0.058	0.168	641
1996	1.341 **	0.440	0.043	0.137	651
1995	1.522 **	-0.111	0.050	0.126	653
1994	1.557 **	-0.105	0.057	0.088	630
1993	1.422 **	-0.711	0.065	0.018	690
1992	1.450 **	-0.870	0.074	-0.020	843
1991	1.537 #	-0.903	0.071	-0.027	783
1990	1.320 **	-0.749	0.055	0.013	732
1990-21	1.351 **	0.096	0.071	0.121	54,331
99% conf.	0.016				

Exhibit 5 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Total Annual Return Residuals & Mean Returns & Number of Observations

Apartment Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2021	1.829 #	1.000	0.092	0.206	1,822
2020	1.723 **	-0.092	0.036	0.023	1,821
2019	1.459 **	0.064	0.029	0.061	1,553
2018	1.584 **	0.275	0.032	0.064	1,412
2017	1.660 **	0.351	0.033	0.066	1,413
2016	1.635 **	0.000	0.042	0.075	1,382
2015	1.712 **	0.688	0.051	0.125	1,316
2014	1.655 **	0.604	0.040	0.105	1,305
2013	1.730 **	0.492	0.039	0.105	1,299
2012	1.623 **	0.289	0.045	0.114	1,341
2011	1.758 **	-0.082	0.060	0.154	1,322
2010	1.780 **	0.277	0.083	0.178	1,250
2009	1.904 **	-0.234	0.073	-0.142	1,313
2008	1.925 **	0.096	0.078	-0.052	1,315
2007	1.419 **	0.532	0.046	0.111	1,091
2006	1.394 **	0.647	0.058	0.161	882
2005	1.291 **	0.867	0.062	0.187	720
2004	1.568 **	0.768	0.049	0.122	705
2003	1.571 **	0.259	0.046	0.089	705
2002	1.748 **	0.434	0.051	0.083	685
2001	1.776 **	-0.092	0.048	0.089	607
2000	1.658 **	0.449	0.045	0.116	558
1999	1.492 **	0.457	0.037	0.112	463
1998	1.592 **	0.744	0.046	0.129	415
1997	1.484 **	0.580	0.041	0.120	406
1996	1.519 **	0.488	0.043	0.114	362
1995	1.520 **	0.758	0.041	0.128	310
1994	1.584 **	0.436	0.059	0.128	298
1993	1.268 **	0.285	0.051	0.109	276
1992	1.232 **	-0.724	0.042	0.028	226
1991	1.503 **	-0.792	0.062	-0.009	197
1990	2.000 #	0.117	0.060	0.067	165
1990-21	1.455 **	0.093	0.060	0.090	28,875
99% conf.	0.023				

Statistically significant confidence of non-Normality $\alpha \neq 2.0$:

** = 99% confidence

= Confidence Interval indeterminate

α is the Characteristic Exponent, and only equals 2.0 for the Normal distribution

β is the Skewness Parameter in the range -1.0 to +1.0

γ is the (positive) Scale Parameter which measures the spread of the distribution about δ

Note: The mean returns are shown in Exhibit 6 simply for purposes of completeness.

Exhibit 6
 Characteristic Exponent “Alpha” of MLE Stable Distribution Parameters for
 NCREIF MVI+FCFY Series by Property Type, 1990 to 2021
 (bands indicate 99% confidence interval)

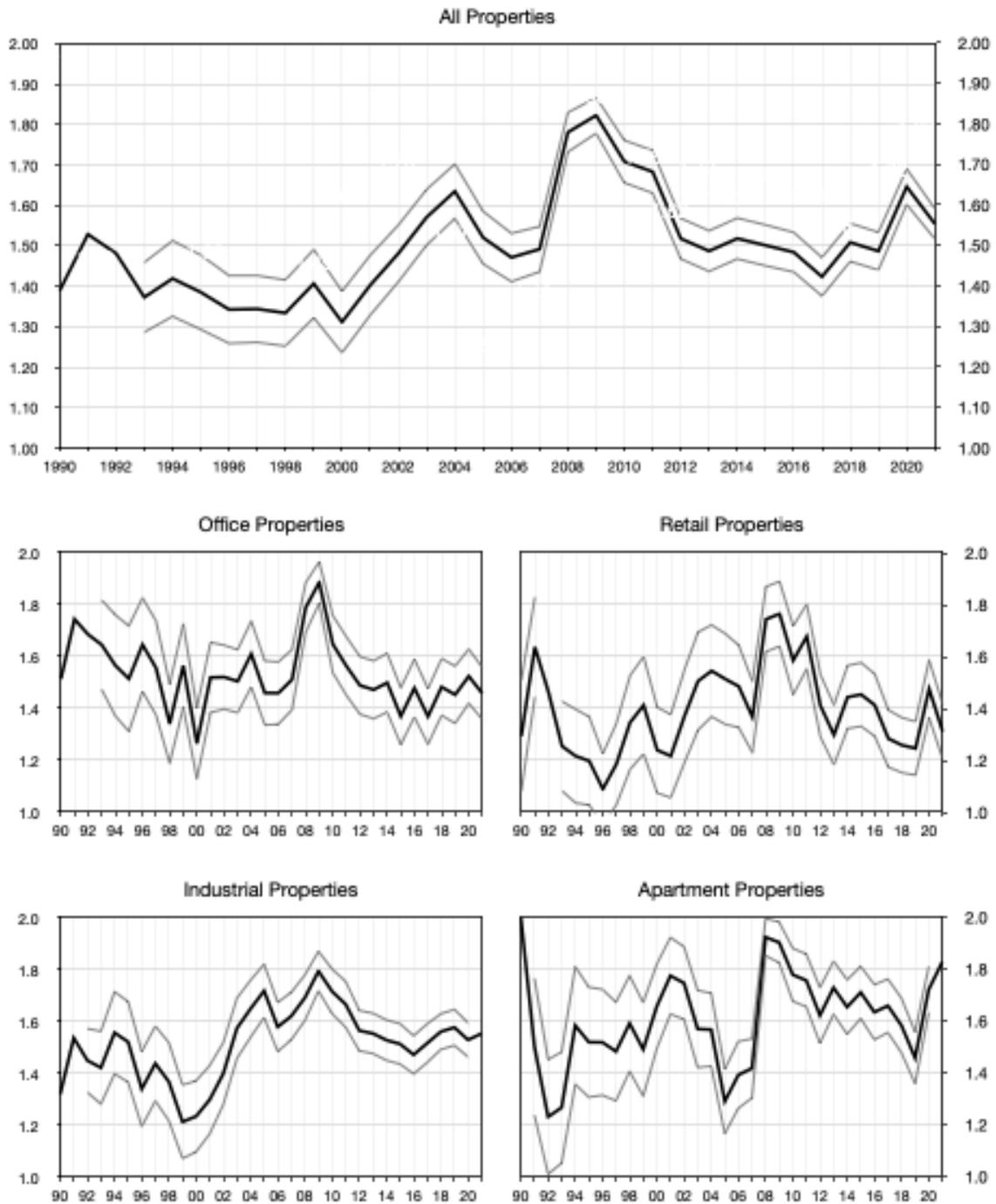


Exhibit 7
 Skewness Parameter “Beta” of MLE Stable Distribution Parameters for NCREIF
 MVI+FCFY Series by Property Type, 1990 to 2021
 (bands indicate 99% confidence interval)

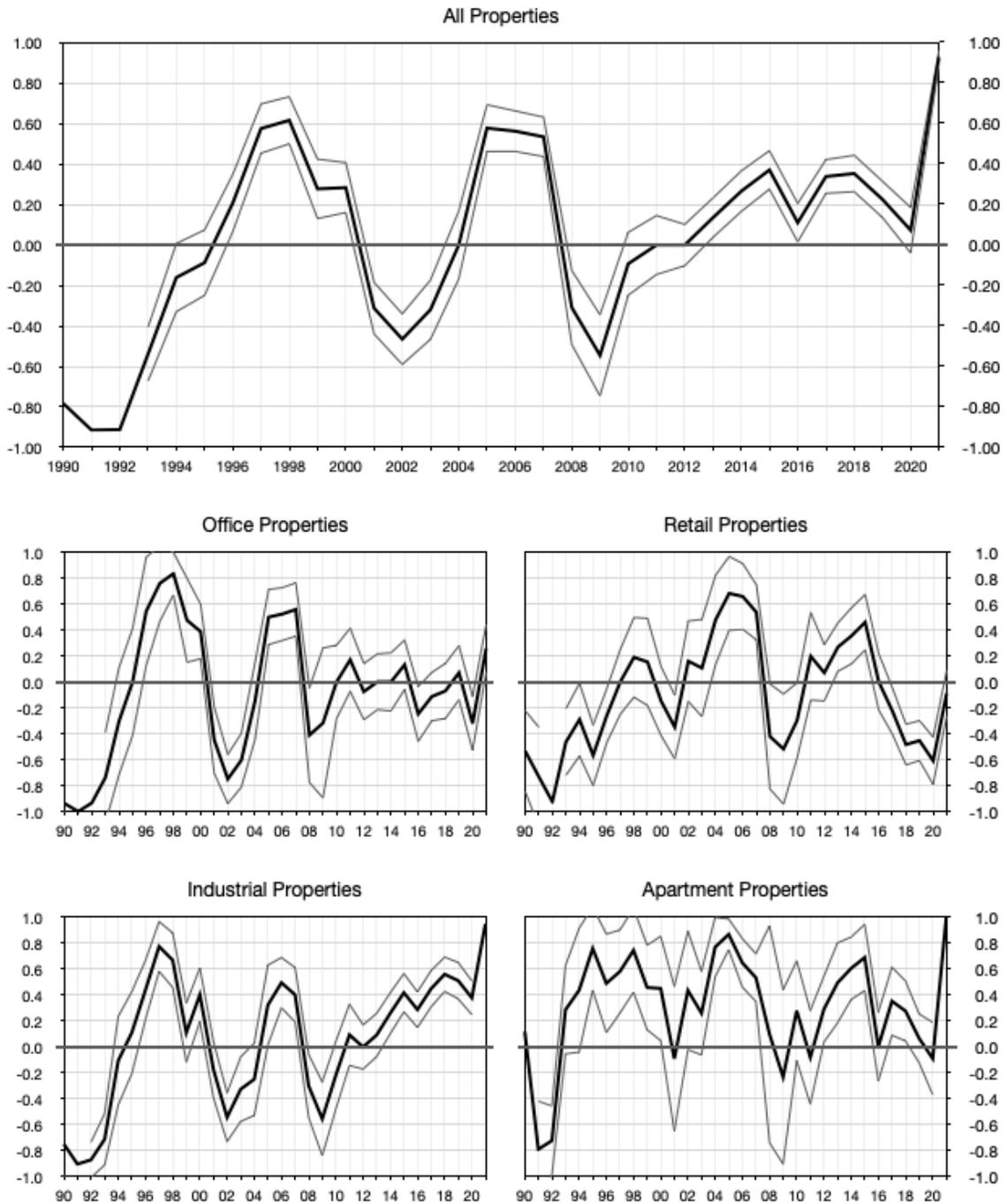


Exhibit 8
 Scale Parameter “Gamma” of MLE Stable Distribution Parameters for NCREIF
 MVI+FCFY Series by Property Type, 1990 to 2021
 (bands indicate 99% confidence interval)

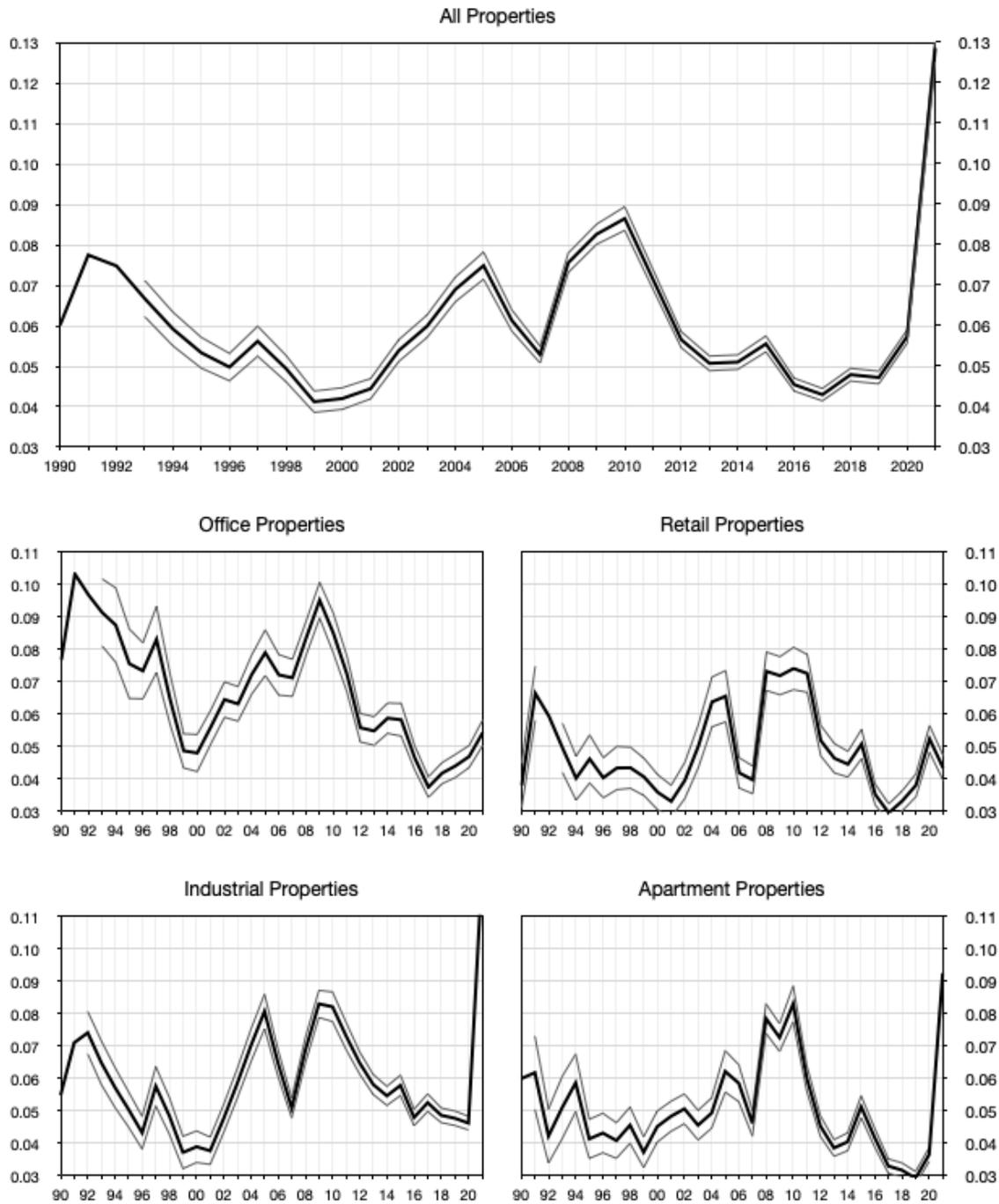


Exhibit 9a
Distribution of Total Annual Return Residuals
NCREIF MVI+FCFY Series, All Properties, 1990 to 2021

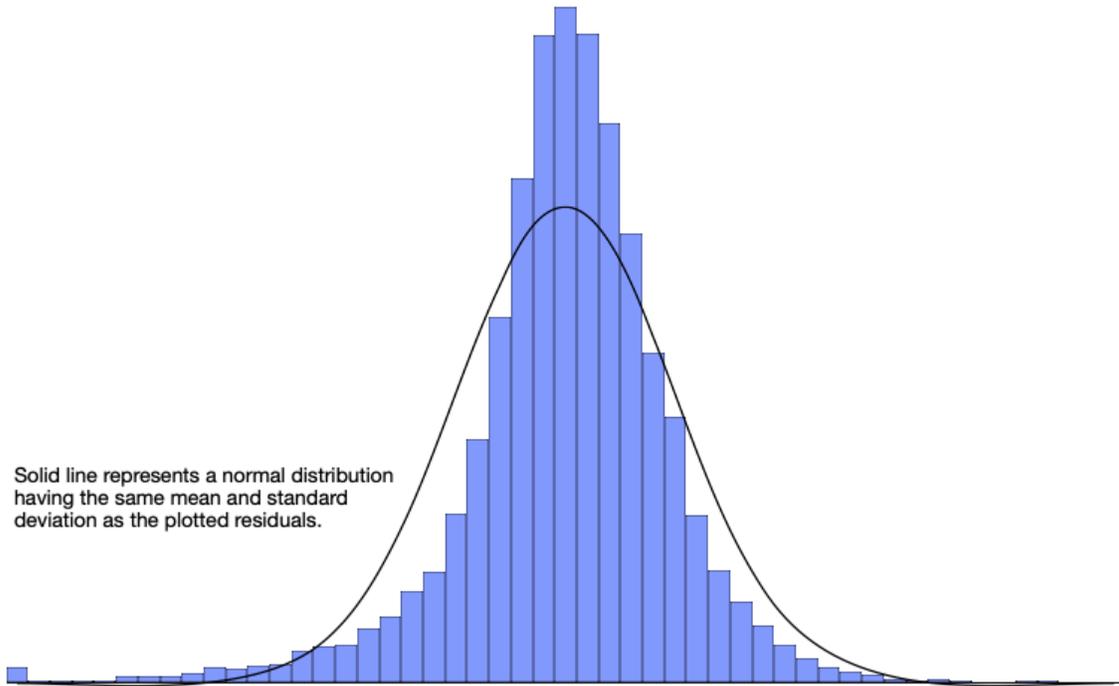


Exhibit 9b
Difference in Frequency, Total Annual Return Residuals to Normal Distribution
NCREIF MVI+FCFY Series, All Properties, 1990 to 2021

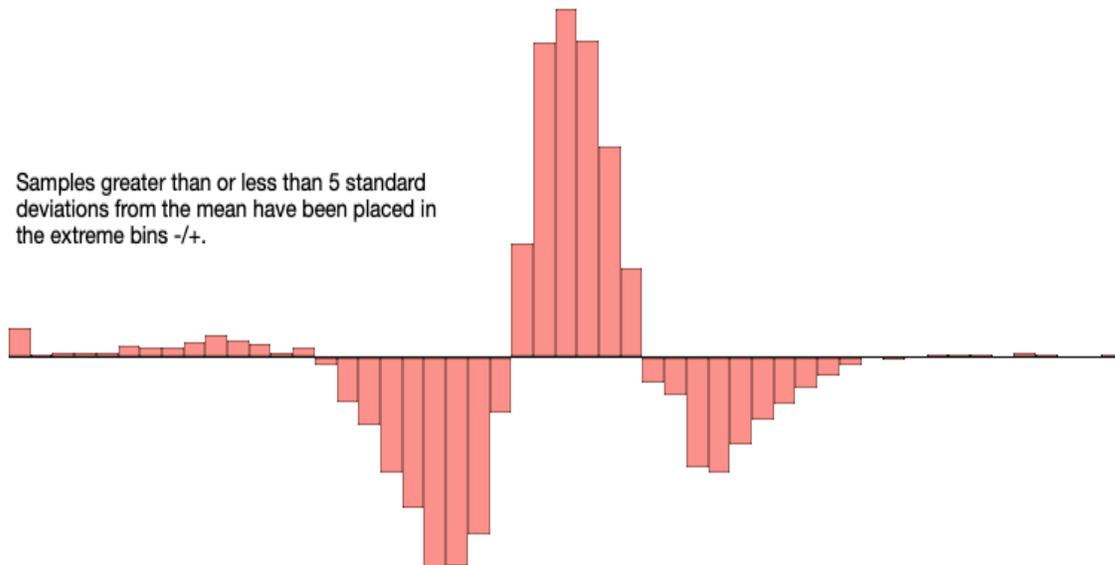


Exhibit 10
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Office Properties:

Year or Period	Std Dev	Semi-IQR	γ	Semi-IQR minus γ	Std Dev minus γ
2021	0.144	0.052	0.054	-0.002	0.090
2020	0.103	0.046	0.047	-0.001	0.093
2019	0.098	0.040	0.044	-0.004	0.056
2018	0.094	0.038	0.042	-0.004	0.054
2017	0.096	0.035	0.037	-0.002	0.059
2016	0.109	0.042	0.046	-0.004	0.063
2015	0.168	0.054	0.058	-0.004	0.110
2014	0.157	0.052	0.059	-0.007	0.098
2013	0.128	0.050	0.055	-0.005	0.073
2012	0.153	0.052	0.056	-0.004	0.097
2011	0.256	0.067	0.072	-0.005	0.184
2010	0.170	0.073	0.085	-0.012	0.085
2009	0.148	0.091	0.095	-0.004	0.053
2008	0.135	0.083	0.083	0.000	0.052
2007	0.147	0.073	0.071	0.002	0.076
2006	0.152	0.073	0.072	0.001	0.080
2005	0.169	0.080	0.079	0.001	0.090
2004	0.137	0.065	0.072	-0.007	0.065
2003	0.126	0.063	0.063	0.000	0.063
2002	0.123	0.067	0.064	0.003	0.059
2001	0.107	0.055	0.056	-0.001	0.051
2000	0.125	0.048	0.048	0.000	0.077
1999	0.095	0.045	0.049	-0.004	0.046
1998	0.157	0.076	0.064	0.012	0.093
1997	0.168	0.089	0.083	0.006	0.085
1996	0.161	0.069	0.073	-0.004	0.088
1995	0.148	0.066	0.075	-0.009	0.073
1994	0.163	0.079	0.087	-0.008	0.076
1993	0.158	0.090	0.091	-0.001	0.069
1992	0.163	0.099	0.097	0.002	0.066
1991	0.163	0.116	0.103	0.013	0.060
1990	0.149	0.085	0.077	0.008	0.072
1990-21	0.167	0.068	0.074	-0.006	0.093

Exhibit 10 (continued)
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Retail Properties:

Year or Period	Std Dev	Semi-IQR	γ	Semi-IQR minus γ	Std Dev minus γ
2021	0.136	0.040	0.043	-0.003	0.093
2020	0.110	0.055	0.052	0.003	0.068
2019	0.121	0.041	0.038	0.003	0.063
2018	0.095	0.036	0.033	0.003	0.062
2017	0.084	0.028	0.029	-0.001	0.055
2016	0.088	0.033	0.035	-0.001	0.053
2015	0.118	0.052	0.051	0.001	0.067
2014	0.101	0.042	0.044	-0.002	0.057
2013	0.121	0.045	0.046	-0.001	0.075
2012	0.123	0.049	0.052	-0.003	0.071
2011	0.134	0.066	0.072	-0.006	0.062
2010	0.203	0.069	0.074	-0.005	0.129
2009	0.117	0.065	0.072	-0.007	0.045
2008	0.191	0.074	0.073	0.001	0.118
2007	0.104	0.042	0.040	0.002	0.064
2006	0.090	0.045	0.042	0.003	0.048
2005	0.152	0.068	0.065	0.003	0.087
2004	0.127	0.064	0.064	0.000	0.063
2003	0.119	0.047	0.050	-0.003	0.069
2002	0.094	0.037	0.039	-0.002	0.055
2001	0.117	0.034	0.033	0.001	0.084
2000	0.117	0.035	0.036	-0.001	0.081
1999	0.090	0.038	0.041	-0.003	0.049
1998	0.107	0.042	0.043	-0.001	0.064
1997	0.135	0.040	0.043	-0.003	0.092
1996	0.135	0.043	0.040	0.003	0.095
1995	0.133	0.050	0.046	0.004	0.089
1994	0.114	0.040	0.040	0.000	0.074
1993	0.141	0.055	0.049	0.006	0.092
1992	0.118	0.076	0.059	0.017	0.059
1991	0.119	0.069	0.066	0.003	0.053
1990	0.097	0.039	0.038	0.001	0.059
1990-21	0.112	0.056	0.061	-0.005	0.051

Exhibit 10 (continued)
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Industrial Properties:

Year or Period	Std Dev	Semi-IQR	γ	Semi-IQR minus γ	Std Dev minus γ
2021	0.214	0.135	0.123	0.012	0.091
2020	0.087	0.045	0.046	0.001	0.041
2019	0.076	0.049	0.048	0.001	0.028
2018	0.091	0.049	0.049	0.000	0.041
2017	0.087	0.051	0.053	-0.002	0.034
2016	0.097	0.046	0.048	-0.002	0.049
2015	0.102	0.059	0.058	0.001	0.044
2014	0.104	0.052	0.055	-0.003	0.049
2013	0.091	0.054	0.058	-0.003	0.033
2012	0.106	0.058	0.065	-0.007	0.041
2011	0.136	0.068	0.073	-0.005	0.063
2010	0.147	0.074	0.082	-0.008	0.065
2009	0.126	0.079	0.083	-0.004	0.043
2008	0.141	0.065	0.069	-0.004	0.072
2007	0.098	0.048	0.051	-0.003	0.047
2006	0.136	0.063	0.065	-0.002	0.071
2005	0.137	0.074	0.081	-0.007	0.056
2004	0.137	0.062	0.070	-0.008	0.067
2003	0.114	0.053	0.059	-0.006	0.055
2002	0.109	0.047	0.048	-0.001	0.061
2001	0.109	0.037	0.038	-0.001	0.071
2000	0.124	0.038	0.039	-0.001	0.085
1999	0.111	0.034	0.037	-0.003	0.074
1998	0.117	0.051	0.049	0.002	0.068
1997	0.137	0.062	0.058	0.004	0.079
1996	0.110	0.043	0.043	0.000	0.067
1995	0.098	0.046	0.050	-0.004	0.048
1994	0.110	0.051	0.057	-0.006	0.054
1993	0.134	0.076	0.065	-0.011	0.069
1992	0.163	0.091	0.074	0.017	0.089
1991	0.132	0.083	0.071	0.012	0.061
1990	0.125	0.065	0.055	0.010	0.070
1990-21	0.172	0.066	0.071	-0.005	0.101

Exhibit 10 (continued)
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Apartment Properties:

Year or Period	Std Dev	Semi-IQR	γ	Semi-IQR minus γ	Std Dev minus γ
2021	0.143	0.098	0.092	0.006	0.051
2020	0.063	0.034	0.036	-0.002	0.027
2019	0.068	0.027	0.029	-0.002	0.039
2018	0.064	0.029	0.032	-0.003	0.032
2017	0.059	0.030	0.033	-0.003	0.026
2016	0.078	0.039	0.042	-0.003	0.036
2015	0.091	0.049	0.051	-0.002	0.040
2014	0.078	0.040	0.040	0.000	0.038
2013	0.069	0.039	0.039	0.000	0.030
2012	0.088	0.044	0.045	-0.001	0.043
2011	0.100	0.058	0.060	-0.002	0.040
2010	0.136	0.080	0.083	-0.003	0.053
2009	0.112	0.067	0.072	-0.005	0.040
2008	0.118	0.077	0.078	-0.001	0.040
2007	0.102	0.048	0.046	0.002	0.056
2006	0.141	0.063	0.058	0.005	0.083
2005	0.174	0.073	0.062	0.011	0.112
2004	0.096	0.052	0.049	0.003	0.047
2003	0.086	0.043	0.046	-0.003	0.040
2002	0.085	0.043	0.051	-0.008	0.034
2001	0.083	0.042	0.048	-0.006	0.035
2000	0.084	0.039	0.045	-0.006	0.039
1999	0.075	0.036	0.037	-0.001	0.038
1998	0.084	0.042	0.046	-0.004	0.038
1997	0.095	0.036	0.041	-0.005	0.054
1996	0.087	0.039	0.043	-0.004	0.084
1995	0.099	0.042	0.041	0.001	0.057
1994	0.112	0.051	0.059	-0.008	0.053
1993	0.124	0.049	0.051	-0.002	0.073
1992	0.096	0.058	0.042	0.004	0.054
1991	0.117	0.070	0.062	0.008	0.055
1990	0.085	0.042	0.060	-0.018	0.025
1990-21	0.125	0.055	0.060	-0.005	0.065

Exhibit 11a
Risk Reduction for Various α versus Number of Assets in Portfolio
Curves Represent α from 2.00 to 1.40 in Increments of 0.10

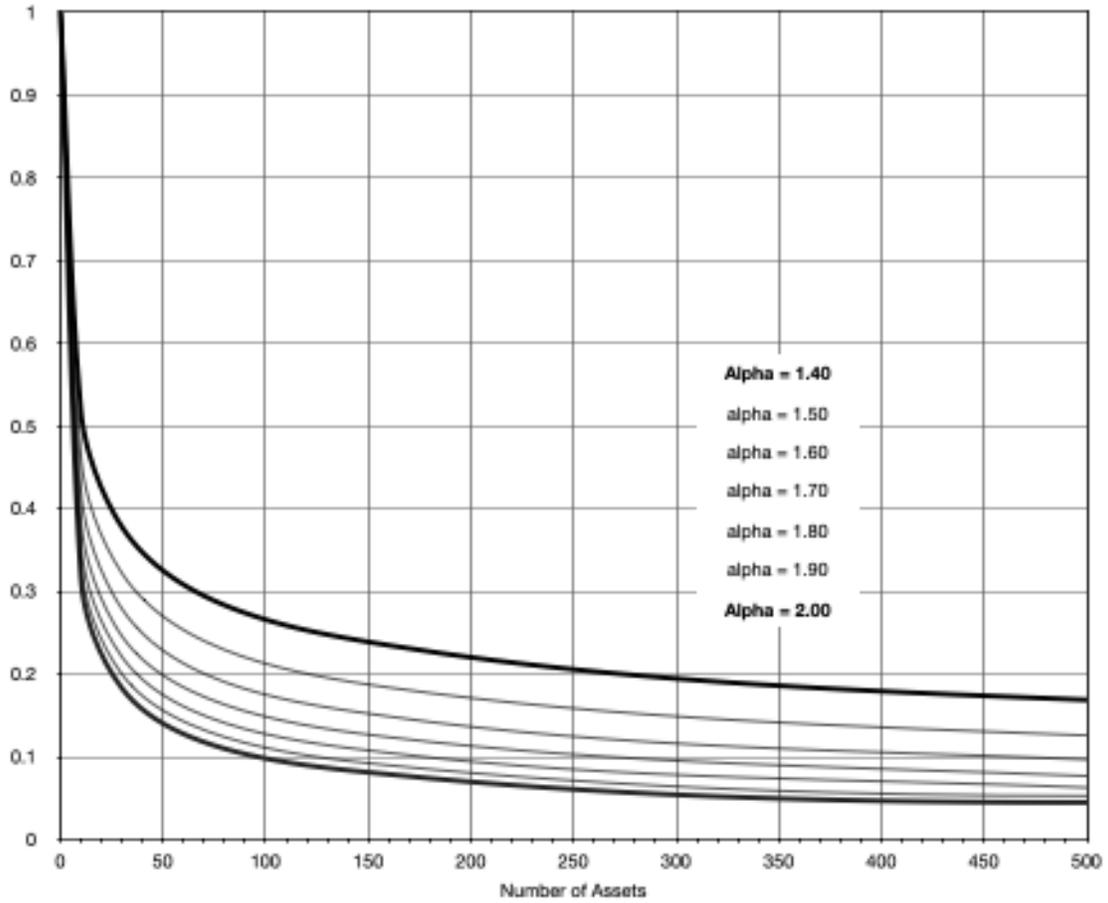
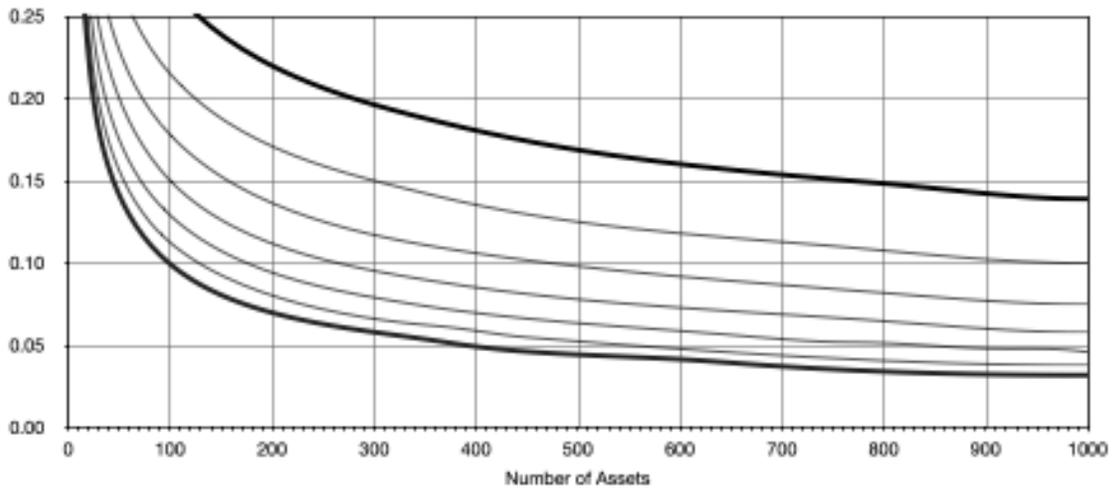


Exhibit 11b
Risk Reduction for Various α versus Number of Assets in Portfolio
Curves Represent α from 2.00 to 1.40 in Increments of 0.10



1 The implementation of other analytical techniques up until the availability of Maximum Likelihood Estimators (MLE) for Levy-stable distributions is related in Young, and Graff (1995).

2 Less frequently there are problems with market value estimates in a quarter such as recording a downpayment as the initial market value followed by the balance of the purchase price as the market value in the subsequent quarter. These cause extreme distortion of quarterly returns for individual properties, but are largely obscured in the aggregate NPI returns commonly cited as representative of the asset class. However, when working with individual property returns or smaller aggregations of property returns as in this study, these problems would necessarily distort the return distribution statistics as they unfortunately did in earlier NPI-based studies.

3 Perhaps it should be noted that there have been other attempts to test the null hypothesis that real estate return distributions are Gaussian Normal using more conventional statistical techniques. The authors know of no cases that resulted in failing to reject the null. For example, there have been studies in the U.S. and even more in the U.K. using Chi-Square, Kolmogorov-Smirnov, or Anderson-Darling tests of common distributions like Logistic, Normal, Student's t, or Extreme Value. For a summary of these studies pre-2000, see Lizieri and Ward (2001).

4 It may be worth noting that the numerators of the Price and Cash Flow formulas are those originally proposed by Young, Geltner, McIntosh, and Poutasse (1995 and 1996) as replacements for the so-called Capital and Income Returns. Since NCREIF did not adopt the changes and retained the original formulation of Capital and Income Returns, the new Price and Cash Flow formulas were introduced for researchers interested in the Young, Geltner, McIntosh, and Poutasse concept. Notice too that the authors also proposed changing the NPI Total Return, Income Return, and Capital Return denominator to simply the beginning quarter's market value.

5 Examples of Partial Sales (PS) include the net sales price of one building from say a multi-building industrial park or the net sales price of an outparcel on the periphery of a shopping center.

6 Capital expenditures are generally reported as positive numbers, but occasionally there will be accounting "reversals" resulting in negative numbers for reported capital expenditures in a particular period. Some reversals may result from journal entries that reclassify or move outlays between periods.

7 Each of these have different risk characteristics per Brown (1998)

8 Passive investment in equity real estate is a fool's errand. Those who think they can invest passively in real estate by buying shares of REITs soon learn they have just bought stock.

9 The astute observer will immediately recognize a paradox in that efficient frontier graphics constitute a parametric plot that requires a covariance matrix. If Levy-stable distributions have no variance, they can have no covariances. One must remember, however, that Levy-stable distributions lack a variance in the limit. All finite samples have a variance that can be calculated. The demonstration illustrates the shape of the "frontier" using samples that are presumed to be drawn from a Levy-stable population having parameters supplied by the user. The demonstration is located at:

<http://demonstrations.wolfram.com/FormingTheEfficientFrontierWhenReturnsAreNonNormal/>.